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FCNC top quark production via anomalous tqV couplings beyond leading order

Nikolaos Kidonakis

Cavendish Laboratory, University of Cambridge
Madingley Road, Cambridge CB3 0HE, U.K.
E-mail: kidonaki@hep.phy.cam.ac.uk

Alexander Belyaev

Physics Department, Florida State University
Tallahassee, FL 32306-4350, U.S.A.
E-mail: belyaev@hep.fsu.edu

Abstract: We calculate flavor-changing neutral-current (FCNC) processes involving top-quark production via anomalous tqV couplings at the Tevatron and HERA colliders. We cover the FCNC processes $p\bar{p} \to tZ$, $p\bar{p} \to t\gamma$, $p\bar{p} \to tt$, and $ep \to et$. We go beyond leading order and include the effects of soft-gluon corrections through next-to-next-to-leading order. We demonstrate the stabilisation of the cross section with respect to the variation of QCD scale, and we investigate the reach of the Tevatron and HERA colliders.

Keywords: Beyond Standard Model, NLO Computations, Hadronic Colliders.

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Contents

1. Introduction

The top quark, the heaviest known fermion, occupies a unique place in the search for new physics beyond the Standard Model (SM). This is the only fermion with a mass of about the scale of the electroweak symmetry breaking (EWSB). Therefore the study of its properties and their possible deviations from SM predictions could shed a light on the mechanism of EWSB. Top quark physics can probe various physics beyond the SM: anomalous gluon-top quark couplings [1], anomalous Wtb couplings [2], new strong dynamics [3], flavor-changing neutral-currents (FCNC) [4], R-parity violating SUSY effects [5], CP-violation effects [6], and effects of Kaluza-Klein excited W-bosons [7].

In particular, the study of FCNC couplings involving the top quark is well motivated. The effective lagrangian involving such couplings of a t, q pair to massless bosons is the following:

$$\Delta\mathcal{L}^{\rm eff} = \frac{1}{\Lambda}\,\kappa_{tqV}\,e\,\bar{t}\,\sigma_{\mu\nu}\,q\,F_V^{\mu\nu} + {\rm h.c.}\,, \tag{1.1}$$

where κ_{tqV} is the anomalous FCNC coupling, with q a u- or c-quark and V a photon or Z-boson; $F_V^{\mu\nu}$ are the usual photon/Z-boson field tensors; $\sigma_{\mu\nu} = (i/2)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$ with γ_μ the Dirac matrices; e is the electron charge; and Λ is an effective scale which we will take throughout this paper to be equal to the top quark mass, which we denote by m. In the SM these operators can be induced through higher-order loops, however these effects are too small to be observable [8].

In some models involving new physics FCNC top-quark couplings could appear at tree-level and, therefore, lead to large FCNC effects. Such enchancements of FCNC couplings involving top quarks could appear in two-Higgs doublet models and supersymmetric models [9] as well as in multiple-Higgs doublet models [10]. Models with FCNC coupled singlet quarks, dynamical electroweak symmetry breaking, or compositeness, can further enchance FCNC top-quark couplings [11].

The presently operating colliders such as the Tevatron and HERA have a nice opportunity to probe FCNC interactions in the top-quark sector. In order to establish accurate limits on FCNC couplings, experiments need accurate predictions of cross sections for FCNC processes. We have shown in [12] that at HERA uncertainties for the tree-level FCNC cross section $ep \to et$ can be very big, as the cross section can vary by a factor of two due to the choice of the QCD scale. This fact unavoidably requires involving the higher-order corrections for the stabilization of the FCNC cross section.

In this paper we study some principal FCNC processes both for the Tevatron and HERA colliders involving $t-u-V$ couplings with V a photon or Z-boson [13]. The effects of $\gamma - Z$ interference are also taken into account in our studies. We include soft-gluon corrections at next-to-leading order (NLO) and next-to-next-to-leading order (NNLO). We present the stabilization of the NNLO cross sections with respect to the variation of the renormalization/factorization scales.

In sections 2 and 3 we study FCNC processes of the associated *top*-quark production with Z-boson and photon at the Tevatron, respectively. Section 4 is devoted to the $uu \to tt$ process of same-sign top quark production at the Tevatron. Section 5 extends our previous study on single *top*-quark production at HERA, and we now consider this process at NNLO level as well as study both $t-u-\gamma$ and $t-u-Z$ vertexes and take into account $\gamma - Z$ interference. Finally in section 6 we draw our conclusions.

We note that contributions involving the charm quark via $t-c-V$ couplings are greatly suppressed because of the small charm parton densities. In each section we make a note about the contribution from charm quarks. We also note that we don't consider $t\bar{t}$ production and tq or $t\bar{q}$ production in this paper because the Standard Model cross sections for these processes overwhelm any FCNC contributions.

In the present paper we use the notation $\hat{\sigma}$ for parton-level differential cross sections, while σ denotes the hadronic cross sections. We also define a kinematical variable s_4, for each subprocess, that goes to zero at threshold. The effects of the soft-gluon radiation appear in the form of logarithmic "plus" distributions with respect to s_4 of the type

$$\left[\frac{\ln^k(s_4/m^2)}{s_4}\right]_+, \qquad k \leq 2n-1\,, \tag{1.2}$$

at n-th order in the QCD coupling α_s. These plus distributions are the remnants of cancellations of infrared divergences between soft and virtual contributions to the cross section. We define the leading logarithms (LL) with $k = 2n-1$ and the next-to-leading logarithms (NLL) with $k = 2n-2$.

The plus distributions are defined by their integral with any smooth function f, such as parton distributions, as

$$\begin{aligned}\int_0^{s_{4\,\max}} ds_4 f(s_4)\left[\frac{\ln^k(s_4/m^2)}{s_4}\right]_+ &= \int_0^{s_{4\,\max}} ds_4 \frac{\ln^k(s_4/m^2)}{s_4}[f(s_4)-f(0)]+\\ &\quad+\frac{1}{k+1}\ln^{k+1}\left(\frac{s_{4\,\max}}{m^2}\right)f(0)\,. \end{aligned}\tag{1.3}$$

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They provide significant and dominant contributions to the cross section near threshold, where there is limited phase space for real gluon emission, as has been shown for many Standard Model processes, including top production [14], direct photon production [15], jet production [16], and W-boson production [17]. A unified approach for the calculation of these soft-gluon corrections for various processes in hadron-hadron and lepton-hadron colliders has recently been presented in ref. [18]. We follow that reference in calculating soft-gluon corrections to FCNC processes at NLO and NNLO with NLL accuracy, i.e. keeping LL and NLL. We work in the $\overline{\mathrm{MS}}$ scheme throughout the paper.

2. $gu \to tZ$

We begin with FCNC tZ production at the Tevatron. For the process $g(p_g) + u(p_u) \to t(p_t) + Z(p_Z)$, we define the kinematical invariants $s = (p_g + p_u)^2$, $t = (p_g - p_t)^2$, $u = (p_u - p_t)^2$, and $s_4 = s + t + u - m^2 - m_Z^2$, where m_Z is the Z-boson mass and m is the top quark mass. Note that near threshold, i.e. when we have just enough partonic energy to produce the tZ final state, $s_4 \to 0$. The respective tree-level Feynman diagrams are shown in figure 1.

The differential Born cross section is

$$\frac{d^2\hat{\sigma}_B^{gu\to tZ}}{dt\,du} = F_B^{gu\to tZ}\,\delta(s_4) \tag{2.1}$$

where

$$\begin{aligned} F_B^{gu\to tZ} = & \frac{2\pi\alpha\alpha_s\kappa_Z^2}{3m^2 s^3 (m^2-t)^2} \times \\ & \times \Big\{ 2m^8 - m^6(3m_Z^2 + 4s + 2t) - \\ & - t\left[2m_Z^6 - 2m_Z^4(s+t) - 4st(s+t) + m_Z^2(s+t)^2\right] + \\ & + m^4\left[2m_Z^4 - m_Z^2(2s+t) + 2(s^2+4st+t^2)\right] + \\ & + m^2\left[2m_Z^6 - 4m_Z^4 t + m_Z^2(s+t)(s+5t) - 2t(3s^2+6st+t^2)\right]\Big\}, \end{aligned} \tag{2.2}$$

where $\alpha = e^2/(4\pi)$ and for brevity we define $\kappa_Z \equiv \kappa_{tuZ}$.

The NLO soft-gluon corrections for $gu \to tZ$ are

$$\frac{d^2\hat{\sigma}^{(1)}_{gu\to tZ}}{dt\,du} = F_B^{gu\to tZ}\,\frac{\alpha_s(\mu_R^2)}{\pi}\left\{c_3^{gu\to tZ}\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ + c_2^{gu\to tZ}\left[\frac{1}{s_4}\right]_+ + c_1^{gu\to tZ}\,\delta(s_4)\right\}. \tag{2.3}$$

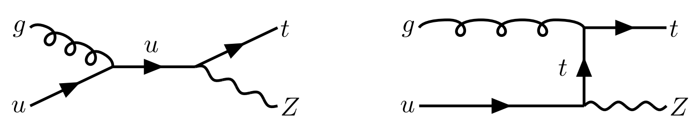

Figure 1: Tree-level Feynman diagrams for the process $gu \to tZ$.

Here $c_3^{gu\to tZ} = 2(C_F + C_A)$, where $C_F = (N_c^2 - 1)/(2N_c)$ and $C_A = N_c$ with $N_c = 3$ the number of colors, and

$$\begin{aligned} c_2^{gu\to tZ} &= 2\mathrm{Re}\Gamma'^{(1)}_S - C_F - C_A - 2C_F \ln\left(\frac{-t+m_Z^2}{m^2}\right) - 2C_A \ln\left(\frac{-u+m_Z^2}{m^2}\right) - \\ &\quad -(C_F + C_A)\ln\left(\frac{\mu_F^2}{s}\right) \\ &\equiv T_2^{gu\to tZ} - (C_F + C_A)\ln\left(\frac{\mu_F^2}{m^2}\right), \end{aligned} \tag{2.4}$$

where μ_F is the factorization scale, and we have defined $T_2^{gu\to tZ}$ as the scale-independent part of $c_2^{gu\to tZ}$. The term $\mathrm{Re}\Gamma'^{(1)}_S$ denotes the real part of the one-loop soft anomalous dimension, which describes noncollinear soft-gluon emission [19]. A one-loop calculation gives

$$\Gamma'^{(1)}_S = C_F \ln\left(\frac{-u+m^2}{m\sqrt{s}}\right) + \frac{C_A}{2}\ln\left(\frac{-t+m^2}{-u+m^2}\right) + \frac{C_A}{2}(1 - \pi i)\,. \tag{2.5}$$

Also

$$c_1^{gu\to tZ} = \left[C_F \ln\left(\frac{-t+m_Z^2}{m^2}\right) + C_A \ln\left(\frac{-u+m_Z^2}{m^2}\right) - \frac{3}{4}C_F - \frac{\beta_0}{4}\right]\ln\left(\frac{\mu_F^2}{m^2}\right) + \frac{\beta_0}{4}\ln\left(\frac{\mu_R^2}{m^2}\right), \tag{2.6}$$

where μ_R is the renormalization scale and $\beta_0 = (11C_A - 2n_f)/3$ is the lowest-order β function, with $n_f = 5$ the number of light quark flavors. Note that $c_1^{gu\to tZ}$ represents the scale-dependent part of the $\delta(s_4)$ corrections. We do not calculate the full virtual corrections here. Our calculation of the soft-gluon corrections includes the leading and next-to-leading logarithms (NLL) and is thus a NLO-NLL calculation.

We next calculate the NNLO soft-gluon corrections for $gu \to tZ$:

$$\begin{aligned} \frac{d^2\hat{\sigma}^{(2)}_{gu\to tZ}}{dt\,du} &= F_B^{gu\to tZ}\frac{\alpha_s^2(\mu_R^2)}{\pi^2} \times \\ &\times\left\{\frac{1}{2}\left(c_3^{gu\to tZ}\right)^2\left[\frac{\ln^3(s_4/m^2)}{s_4}\right]_+ + \right. \\ &\quad + \left[\frac{3}{2}c_3^{gu\to tZ}\,c_2^{gu\to tZ} - \frac{\beta_0}{4}c_3^{gu\to tZ}\right]\left[\frac{\ln^2(s_4/m^2)}{s_4}\right]_+ + \\ &\quad + \left[c_3^{gu\to tZ}\,c_1^{gu\to tZ} + (C_F + C_A)^2\ln^2\left(\frac{\mu_F^2}{m^2}\right) - \right. \\ &\qquad - 2(C_F + C_A)T_2^{gu\to tZ}\ln\left(\frac{\mu_F^2}{m^2}\right) + \\ &\qquad \left. + \frac{\beta_0}{4}c_3^{gu\to tZ}\ln\left(\frac{\mu_R^2}{m^2}\right) - \zeta_2\left(c_3^{gu\to tZ}\right)^2\right]\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ + \\ &\qquad + \left[-(C_F + C_A)\ln\left(\frac{\mu_F^2}{m^2}\right)c_1^{gu\to tZ} - \right. \\ &\qquad\quad - \frac{\beta_0}{4}(C_F + C_A)\ln\left(\frac{\mu_F^2}{m^2}\right)\ln\left(\frac{\mu_R^2}{m^2}\right) + (C_F + C_A)\frac{\beta_0}{8}\ln^2\left(\frac{\mu_F^2}{m^2}\right) - \end{aligned}$$

$$- \zeta_2\, c_2^{gu\to tZ}\, c_3^{gu\to tZ} + \zeta_3\, \left(c_3^{gu\to tZ}\right)^2 \Big] \left[\frac{1}{s_4}\right]_+ \Big\}\,, \tag{2.7}$$

where $\zeta_2 = \pi^2/6$ and $\zeta_3 = 1.2020569\ldots$. We note that only the leading and next-to-leading logarithms are complete. Hence this is a NNLO-NLL calculation. Consistent with a NLL calculation we have also kept all logarithms of the factorization and renormalization scales in the $[\ln(s_4/m^2)/s_4]_+$ terms, and squares of scale logarithms in the $[1/s_4]_+$ terms, as well as ζ_2 and ζ_3 terms that arise in the calculation of the soft corrections. For relevant details in the related process of Standard Model top production see ref. [14].

We now convolute the partonic cross sections with parton distribution functions (PDF) to obtain the hadronic cross section. For the FCNC hadronic cross section $p(p_a) + \bar{p}(p_b) \to t(p_t) + Z(p_Z)$ we define $S = (p_a + p_b)^2$, $T = (p_a - p_t)^2$, and $U = (p_b - p_t)^2$, and note that $p_g = x_a p_a$, $p_u = x_b p_b$, where x denotes the momentum fraction of the hadron carried by the parton. The hadronic cross section is then given by

$$\sigma^{FCNC}_{p\bar{p}\to tZ}(S) = \int_{T_{\min}}^{T_{\max}} dT \int_{-S-T+m^2+m_Z^2}^{m^2+m^2 S/(T-m^2)} dU \int_{(m_Z^2-T)/(S+U-m^2)}^{1} dx_b \times$$
$$\times \int_0^{x_b(S+U-m^2)+T-m_Z^2} ds_4 \frac{x_a x_b}{x_b S + T - m^2}\, \phi(x_a)\, \phi(x_b)\, \frac{d^2\hat{\sigma}_{gu\to tZ}}{dt\, du} \tag{2.8}$$

where

$$T_{\substack{\max\\ \min}} = -\frac{1}{2}(S - m^2 - m_Z^2) \pm \frac{1}{2}\sqrt{(S + m^2 - m_Z^2)^2 - 4m^2 S}\,, \tag{2.9}$$

$x_a = \left[s_4 - m^2 + m_Z^2 - x_b(U - m^2)\right] / (x_b S + T - m^2)$, and $\phi(x)$ are the parton distributions.

In our calculations we are using the MRST2002 NNLO parton distribution functions [20] with the respective three-loop evaluation of α_s. For all three – Born, NLO-NLL, and NNLO-NLL – we use the same MRST2002 NNLO PDF. We also use $\mu_F = \mu_R$ for our numerical results. For reference point we choose κ_Z=0.1 and $\mu \equiv \mu_F = \mu_R = m = 175\,\mathrm{GeV}$. In our paper we quote the theoretical error related to the conventional variation of the scale between $m/2$ and $2m$.

We present our numerical results in figure 2. The left frame presents the cross section for the process $gu \to tZ$ versus top-quark mass at the Tevatron with $\sqrt{S} = 1.96$ TeV, at Born, NLO-NLL, and NNLO-NLL orders. Note that the first-order corrections, $\Delta\sigma_{\text{NLO-NLL}}$, and the second-order corrections, $\Delta\sigma_{\text{NNLO-NLL}}$, are positive for $\mu = m$. There is clear stabilisation of the cross section due to the $\Delta\sigma_{\text{NLO-NLL}}$ and $\Delta\sigma_{\text{NNLO-NLL}}$ corrections over a large range of μ/m presented in the right frame of figure 2. At the reference point, $\mu = m = 175\,\mathrm{GeV}$ and $\kappa_Z = 0.1$, we have the following results:

$$\sigma^{gu\to tZ}_{\text{Born}} = 61\,\mathrm{fb},\ \Delta\sigma^{gu\to tZ}_{\text{NLO-NLL}} = 20\,\mathrm{fb},\ \Delta\sigma^{gu\to tZ}_{\text{NNLO-NLL}} = 6\,\mathrm{fb}\,.$$

Taking into account the theoretical error due to the scale variation between $m/2$ and $2m$ one thus has for the total NNLO-NLL cross section:

$$\sigma^{gu\to tZ}_{\text{NNLO-NLL}} = 87^{+2}_{-3}\,\mathrm{fb}\,.$$

Since the total cross section is proportional to κ_Z^2 it is straightforward to rescale it for any given value of the anomalous FCNC coupling.

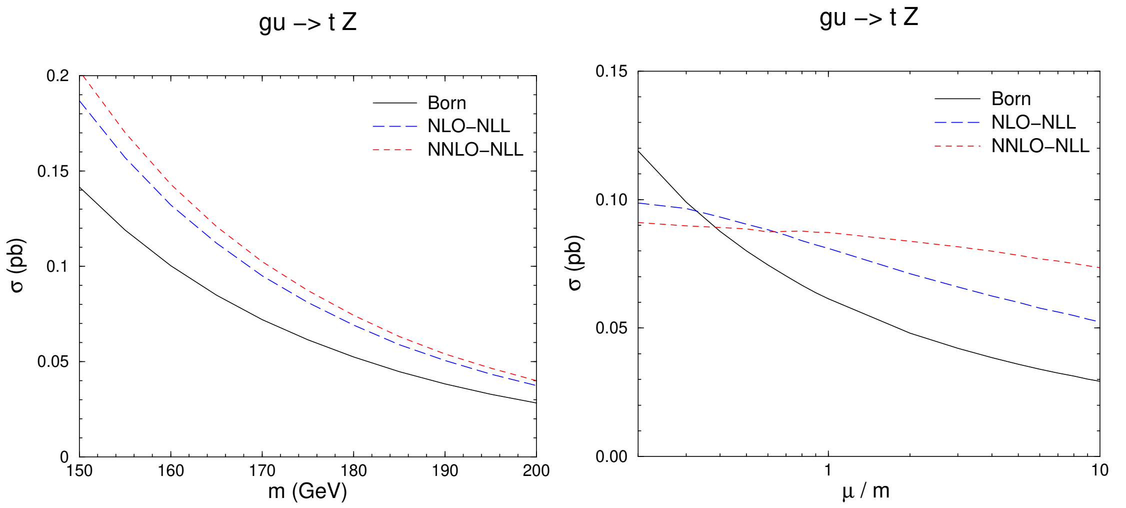

Figure 2: The Born, NLO-NLL, and NNLO-NLL cross sections for $gu \to tZ$ in $p\bar{p}$ collisions with $\sqrt{S} = 1.96\,\mathrm{TeV}$ and $\kappa_Z = 0.1$. Left: cross section versus top-quark mass at scale $\mu = m$; right: cross section versus the scale for $m = 175\,\mathrm{GeV}$.

For completeness we would like also to quote the cross section for the case of κ_{tcZ} coupling. Since the initial c-quark is coming from the sea, and thus the charm density is quite small, the total cross section for $gc \to tZ$ at the Tevatron, with $\mu = m = 175\,\mathrm{GeV}$ and assuming the anomalous coupling is again 0.1, is about 40 times smaller: $\sigma^{gc\to tZ}_{\text{NNLO-NLL}} = 2.4\,\mathrm{fb}$.

One should also notice that we present all cross sections for *top*-quark production. For experimental studies one should also include the contribution from *anti-top*-quark production which will double the total cross section for all processes at the Tevatron we study here.

3. $gu \to t\gamma$

We continue with FCNC $t\gamma$ production at the Tevatron. For the process $g(p_g) + u(p_u) \to t(p_t) + \gamma(p_\gamma)$, we define the kinematical invariants $s = (p_g + p_u)^2$, $t = (p_g - p_t)^2$, $u = (p_u - p_t)^2$, and $s_4 = s + t + u - m^2$, where again m is the top-quark mass. Note that near threshold $s_4 \to 0$. The respective tree-level Feynman diagrams are shown in figure 3.

The differential Born cross section is

$$\frac{d^2\hat{\sigma}_B^{gu\to t\gamma}}{dt\,du} = F_B^{gu\to t\gamma}\,\delta(s_4) \qquad (3.1)$$

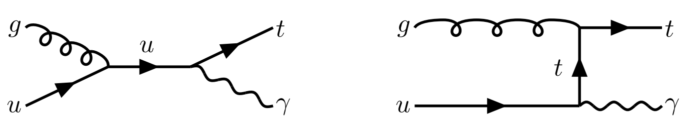

Figure 3: Tree-level Feynman diagrams for the process $gu \to t\gamma$.

where

$$F_B^{gu\to t\gamma}=\frac{4\pi\alpha\alpha_s\kappa_\gamma^2(m^2-s-t)\left[m^6-m^4s-2st^2+m^2t(3s+t)\right]}{3\,m^2\,s^3\,(m^2-t)^2}\,, \tag{3.2}$$

where we define $\kappa_\gamma \equiv \kappa_{tq\gamma}$.

The NLO-NLL corrections for $gu \to t\gamma$ are

$$\frac{d^2\hat{\sigma}^{(1)}_{gu\to t\gamma}}{dt\,du}=F_B^{gu\to t\gamma}\frac{\alpha_s(\mu_R^2)}{\pi}\left\{c_3^{gu\to t\gamma}\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ + c_2^{gu\to t\gamma}\left[\frac{1}{s_4}\right]_+ + c_1^{gu\to t\gamma}\delta(s_4)\right\}. \tag{3.3}$$

Here $c_3^{gu\to t\gamma}=2(C_F+C_A)$,

$$\begin{aligned}c_2^{gu\to t\gamma} &= 2\mathrm{Re}\Gamma'^{(1)}_S - C_F - C_A - 2C_F\ln\left(\frac{-t}{m^2}\right) - 2C_A\ln\left(\frac{-u}{m^2}\right) - (C_F+C_A)\ln\left(\frac{\mu_F^2}{s}\right)\\ &\equiv T_2^{gu\to t\gamma} - (C_F+C_A)\ln\left(\frac{\mu_F^2}{m^2}\right),\end{aligned} \tag{3.4}$$

with $\Gamma'^{(1)}_S$ defined in eq. (2.5), and

$$c_1^{gu\to t\gamma}=\left[C_F\ln\left(\frac{-t}{m^2}\right)+C_A\ln\left(\frac{-u}{m^2}\right)-\frac{3}{4}C_F-\frac{\beta_0}{4}\right]\ln\left(\frac{\mu_F^2}{m^2}\right)+\frac{\beta_0}{4}\ln\left(\frac{\mu_R^2}{m^2}\right). \tag{3.5}$$

We note that the c_i coefficients are similar to those for the $gu \to tZ$ process. This is because the QCD content of the two processes is the same.

The NNLO-NLL corrections for $gu \to t\gamma$ are

$$\begin{aligned}\frac{d^2\hat{\sigma}^{(2)}_{gu\to t\gamma}}{dt\,du} &= F_B^{gu\to t\gamma}\frac{\alpha_s^2(\mu_R^2)}{\pi^2}\times\\ &\times\left\{\frac{1}{2}\left(c_3^{gu\to t\gamma}\right)^2\left[\frac{\ln^3(s_4/m^2)}{s_4}\right]_+ +\right.\\ &+\left[\frac{3}{2}c_3^{gu\to t\gamma}\,c_2^{gu\to t\gamma}-\frac{\beta_0}{4}c_3^{gu\to t\gamma}\right]\left[\frac{\ln^2(s_4/m^2)}{s_4}\right]_+ +\\ &+\left[c_3^{gu\to t\gamma}\,c_1^{gu\to t\gamma}+(C_F+C_A)^2\ln^2\left(\frac{\mu_F^2}{m^2}\right)-2(C_F+C_A)T_2^{gu\to t\gamma}\times\right.\\ &\quad\left.\times\ln\left(\frac{\mu_F^2}{m^2}\right)+\frac{\beta_0}{4}c_3^{gu\to t\gamma}\ln\left(\frac{\mu_R^2}{m^2}\right)-\zeta_2\left(c_3^{gu\to t\gamma}\right)^2\right]\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ +\\ &+\left[-(C_F+C_A)\ln\left(\frac{\mu_F^2}{m^2}\right)c_1^{gu\to t\gamma}-\right.\\ &\quad-\frac{\beta_0}{4}(C_F+C_A)\ln\left(\frac{\mu_F^2}{m^2}\right)\ln\left(\frac{\mu_R^2}{m^2}\right)+(C_F+C_A)\frac{\beta_0}{8}\ln^2\left(\frac{\mu_F^2}{m^2}\right)-\\ &\quad\left.\left.-\zeta_2\,c_2^{gu\to t\gamma}\,c_3^{gu\to t\gamma}+\zeta_3\left(c_3^{gu\to t\gamma}\right)^2\right]\left[\frac{1}{s_4}\right]_+\right\}.\end{aligned} \tag{3.6}$$

Again, they are of the same form as for the $gu \to tZ$ process.

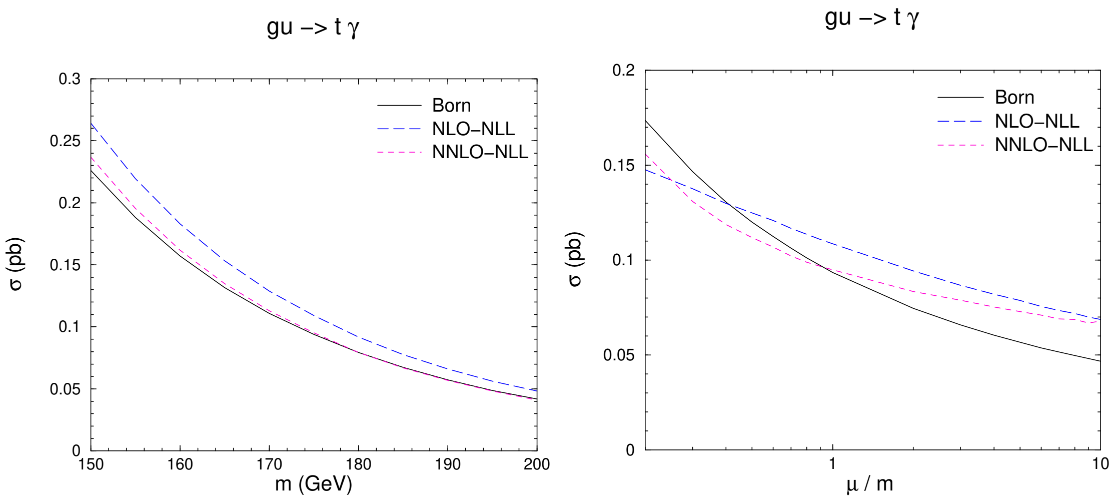

Figure 4: The Born, NLO-NLL, and NNLO-NLL cross sections for $gu \to t\gamma$ in $p\bar{p}$ collisions with $\sqrt{S} = 1.96\,\text{TeV}$ and $\kappa_\gamma = 0.1$. Left: cross section versus top-quark mass at scale $\mu = m$; right: cross section versus the scale for $m = 175\,\text{GeV}$.

For the FCNC hadronic cross section $p(p_a) + \bar{p}(p_b) \to t(p_t) + \gamma(p_\gamma)$ we define $S = (p_a + p_b)^2$, $T = (p_a - p_t)^2$, and $U = (p_b - p_t)^2$, and note that $p_g = x_a p_a$, $p_u = x_b p_b$. The hadronic cross section is then given by

$$\sigma^{FCNC}_{p\bar{p}\to t\gamma}(S) = \int_{m^2-S}^{0} dT \int_{-S-T+m^2}^{m^2+m^2S/(T-m^2)} dU \int_{-T/(S+U-m^2)}^{1} dx_b \int_0^{x_b(S+U-m^2)+T} ds_4 \times$$
$$\times \frac{x_a x_b}{x_b S + T - m^2}\, \phi(x_a)\, \phi(x_b)\, \frac{d^2\hat{\sigma}_{gu\to t\gamma}}{dt\, du} \tag{3.7}$$

with $x_a = \left[s_4 - m^2 - x_b(U - m^2)\right] / (x_b S + T - m^2)$.

Our numerical results are presented in figure 4. The left frame presents the cross section for the process $gu \to t\gamma$ versus top-quark mass at the Tevatron, with $\sqrt{S} = 1.96\,\text{TeV}$, at Born, NLO-NLL, and NNLO-NLL orders. One can see that the NLO-NLL correction is positive at $\mu = m$ but the NNLO-NLL correction is negative. The right frame presents the scale dependence of the cross section which again is stabilized when corrections are added. At $\mu = m = 175\,\text{GeV}$, the Born and NNLO-NLL results happen to be very close to each other because of the negative NNLO-NLL correction. For our reference point, $\mu = m = 175\,\text{GeV}$, $\kappa_\gamma = 0.1$, we have:

$$\sigma^{gu\to t\gamma}_{\text{Born}} = 94\,\text{fb}\,, \qquad \Delta\sigma^{gu\to t\gamma}_{\text{NLO-NLL}} = 15\,\text{fb}\,, \qquad \Delta\sigma^{gu\to t\gamma}_{\text{NNLO-NLL}} = -14\,\text{fb}\,.$$

This gives a NNLO-NLL total cross section

$$\sigma^{gu\to t\gamma}_{\text{NNLO-NLL}} = 95^{+17}_{-11}\,\text{fb}$$

with theoretical error due to the scale variation between $m/2$ and $2m$.

Since the total cross section is proportional to κ_γ^2 it is straightforward to rescale it for any given value of the anomalous FCNC coupling.

For completeness we would like also to quote the cross section for the case of $\kappa_{tc\gamma}$ coupling. Since the initial c-quark is coming from the sea, the total cross section at the Tevatron for the process $gc \to t\gamma$, at $\mu = m = 175\,\mathrm{GeV}$ and assuming the coupling is again 0.1, is about 35 times lower then the one for $gu \to t\gamma$: $\sigma^{gc\to t\gamma}_{\text{NNLO-NLL}} = 2.7\,\mathrm{fb}$.

Again we note that we present all cross sections for *top*-quark production. For experimental studies one should also include the contribution from *anti-top*-quark production which will double the total cross section at the Tevatron.

4. $uu \to tt$

We now turn to FCNC same-sign top-quark production at the Tevatron. For the process, $u(p_{u_a}) + u(p_{u_b}) \to t(p_1) + t(p_2)$, we define the kinematical invariants $s = (p_{u_a} + p_{u_b})^2$, $t = (p_{u_a} - p_1)^2$, $t_1 = t - m^2$, $u = (p_{u_b} - p_1)^2$, $u_1 = u - m^2$, and $s_4 = s + t_1 + u_1$, with m the top-quark mass. At threshold $s_4 \to 0$. The respective tree-level Feynman diagrams are shown in figure 5.

The color structure of the hard scattering is somewhat complex since we have two top quarks in the final state and two up quarks in the initial state. This is in contrast to all the other processes discussed in this paper which have simple color structure. We thus decompose the tree-level amplitudes in terms of color tensors using the methods and techniques detailed in ref. [19]. As we will see the soft anomalous dimension is now a matrix in color space.

For the process at hand we choose the color basis consisting of singlet exchange in the t and u channels, $c_1 = \delta_{a1}\delta_{b2}$ and $c_2 = \delta_{a2}\delta_{b1}$, where a (b) is the color index for the u-quark with momentum p_{u_a} (p_{u_b}) and 1 (2) is the color index for the top quark with momentum p_1 (p_2). Of course the physical cross section is independent of the specific choice of color basis. We write the Born cross section as a trace of the product of a "hard" function, H, that describes the short-distance hard-scattering, and a "soft" function, S, that describes noncollinear soft gluon emission [14, 18, 19]. Both H and S are 2×2 matrices in the color basis c_1, c_2.

The hard matrix at lowest order, calculated by projecting the tree-level amplitude onto the color basis, is given by

$$H^{(0)} = \frac{1}{16\pi s^2}\begin{bmatrix} H_{11}^{(0)} & H_{12}^{(0)} \\ H_{21}^{(0)} & H_{22}^{(0)} \end{bmatrix} \tag{4.1}$$

with

$$H_{11}^{(0)} = \frac{8}{9}\frac{(4\pi\alpha)^2}{m^4}\left[2m^8 + 4m^6 t + t^2(t+2u)^2 - m^4 t(t+4u) - 2m^2 t^2(t+4u)\right] \times$$

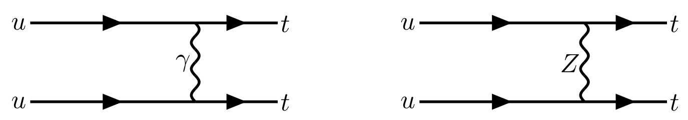

Figure 5: Tree-level Feynman diagrams for the process $uu \to tt$.

$$\times\left[\frac{\kappa_\gamma^4}{t^2}-\frac{2\kappa_\gamma^2\kappa_Z^2}{t(m_Z^2-t)}+\frac{\kappa_Z^4}{(m_Z^2-t)^2}\right] \tag{4.2}$$

and

$$H_{22}^{(0)}=H_{11}^{(0)}(t\leftrightarrow u)\,. \tag{4.3}$$

Here $\kappa_\gamma \equiv \kappa_{tu\gamma}$ and $\kappa_Z \equiv \kappa_{tuZ}$. Also

$$\begin{aligned} H_{12}^{(0)} &= -\frac{4}{9}\frac{(4\pi\alpha)^2}{m^4}\left[8m^8-5m^4tu-2m^6(t+u)+5m^2tu(t+u)-tu(2t+u)(t+2u)\right]\times \\ &\times\left[\frac{\kappa_\gamma^4}{tu}-\frac{2\kappa_\gamma^2\kappa_Z^2}{t(m_Z^2-u)}+\frac{\kappa_Z^4}{(m_Z^2-t)(m_Z^2-u)}\right] \end{aligned} \tag{4.4}$$

and

$$H_{21}^{(0)}=H_{12}^{(0)}(t\leftrightarrow u)\,. \tag{4.5}$$

The soft matrix at lowest order is given by

$$S^{(0)}=\begin{bmatrix} N_c^2 & N_c \\ N_c & N_c^2 \end{bmatrix}. \tag{4.6}$$

Note that the matrix elements of $S^{(0)}$ are simply given by $S_{ji}^{(0)}=\mathrm{Tr}[c_j^* c_i]$.

Then the differential Born cross section is

$$\frac{d^2\hat\sigma_B^{uu\to tt}}{dt\,du}=F_B^{uu\to tt}\delta(s_4) \tag{4.7}$$

with

$$F_B^{uu\to tt}=\mathrm{Tr}\left[H^{(0)}S^{(0)}\right]. \tag{4.8}$$

The NLO-NLL corrections for $uu\to tt$ are

$$\begin{aligned} \frac{d^2\hat\sigma^{(1)}_{uu\to tt}}{dt\,du} &= F_B^{uu\to tt}\frac{\alpha_s(\mu_R^2)}{\pi}\left\{c_3^{uu\to tt}\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ + c_2^{uu\to tt}\left[\frac{1}{s_4}\right]_+ + c_1^{uu\to tt}\delta(s_4)\right\}+ \\ &+\frac{\alpha_s(\mu_R^2)}{\pi}A^{uu\to tt}\left[\frac{1}{s_4}\right]_+ . \end{aligned} \tag{4.9}$$

Here $c_3^{uu\to tt}=4C_F$,

$$c_2^{uu\to tt}=-2C_F-2C_F\ln\left(\frac{t_1u_1}{m^4}\right)-2C_F\ln\left(\frac{\mu_F^2}{s}\right)\equiv T_2^{uu\to tt}-2C_F\ln\left(\frac{\mu_F^2}{m^2}\right), \tag{4.10}$$

where $T_2^{uu\to tt}$ denotes the scale-independent part of $c_2^{uu\to tt}$, and

$$c_1^{uu\to tt}=C_F\left[\ln\left(\frac{t_1u_1}{m^4}\right)-\frac{3}{2}\right]\ln\left(\frac{\mu_F^2}{m^2}\right) \tag{4.11}$$

is again the scale-dependent part of the $\delta(s_4)$ terms. Also

$$A^{uu\to tt}=\mathrm{Tr}\left[H^{(0)}{\Gamma'}_S^{(1)\,\dagger}S^{(0)}+H^{(0)}S^{(0)}{\Gamma'}_S^{(1)}\right] \tag{4.12}$$

is the part of the NLO-NLL corrections not proportional to the Born cross section; this is again because of the complex color structure of the hard scattering. Here ${\Gamma'}_S^{(1)}$ is a 2×2 soft anomalous dimension matrix, calculated at one loop, with elements

$$
\begin{aligned}
{\Gamma'}_{11}^{(1)} &= C_F \left[2 \ln\left(\frac{-t_1}{s}\right) - \ln\left(\frac{m^2}{s}\right)\right] - \frac{1}{2N_c}\left[2 \ln\left(\frac{-u_1}{s}\right) - \ln\left(\frac{m^2}{s}\right) + L_\beta + \pi i\right], \\
{\Gamma'}_{12}^{(1)} &= \ln\left(\frac{-t_1}{s}\right) - \frac{1}{2}\ln\left(\frac{m^2}{s}\right) + \frac{1}{2}L_\beta + \frac{\pi i}{2}, \\
{\Gamma'}_{21}^{(1)} &= \ln\left(\frac{-u_1}{s}\right) - \frac{1}{2}\ln\left(\frac{m^2}{s}\right) + \frac{1}{2}L_\beta + \frac{\pi i}{2}, \\
{\Gamma'}_{22}^{(1)} &= C_F \left[2 \ln\left(\frac{-u_1}{s}\right) - \ln\left(\frac{m^2}{s}\right)\right] - \frac{1}{2N_c}\left[2 \ln\left(\frac{-t_1}{s}\right) - \ln\left(\frac{m^2}{s}\right) + L_\beta + \pi i\right],
\end{aligned}
\tag{4.13}
$$

where $L_\beta = [(1-2m^2/s)/\beta][\ln((1-\beta)/(1+\beta)) + \pi i]$, with $\beta = \sqrt{1-4m^2/s}$. Note that all imaginary parts cancel out in eq. (4.12).

The NNLO-NLL corrections for $uu \to tt$ are

$$
\begin{aligned}
\frac{d^2\hat{\sigma}^{(2)}_{uu\to tt}}{dt\,du} = {} & F_B^{uu\to tt} \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \times \\
& \times \left\{ \frac{1}{2}\left(c_3^{uu\to tt}\right)^2 \left[\frac{\ln^3(s_4/m^2)}{s_4}\right]_+ + \right. \\
& + \left[\frac{3}{2} c_3^{uu\to tt}\, c_2^{uu\to tt} - \frac{\beta_0}{4} c_3^{uu\to tt}\right] \left[\frac{\ln^2(s_4/m^2)}{s_4}\right]_+ + \\
& + \left[c_3^{uu\to tt}\, c_1^{uu\to tt} + 4C_F^2 \ln^2\left(\frac{\mu_F^2}{m^2}\right) - 4C_F T_2^{uu\to tt} \ln\left(\frac{\mu_F^2}{m^2}\right) + \right. \\
& \left. + \frac{\beta_0}{4} c_3^{uu\to tt} \ln\left(\frac{\mu_R^2}{m^2}\right) - \zeta_2 \left(c_3^{uu\to tt}\right)^2 \right] \left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ + \\
& + \left[-2C_F \ln\left(\frac{\mu_F^2}{m^2}\right) c_1^{uu\to tt} - \frac{\beta_0}{2} C_F \ln\left(\frac{\mu_F^2}{m^2}\right) \ln\left(\frac{\mu_R^2}{m^2}\right) + \right. \\
& \left. \left. + C_F \frac{\beta_0}{4} \ln^2\left(\frac{\mu_F^2}{m^2}\right) - \zeta_2\, c_2^{uu\to t\gamma}\, c_3^{uu\to t\gamma} + \zeta_3 \left(c_3^{uu\to t\gamma}\right)^2 \right] \left[\frac{1}{s_4}\right]_+ \right\} + \\
& + \frac{\alpha_s^2(\mu_R^2)}{\pi^2} A^{uu\to tt} \left\{ \frac{3}{2} c_3^{uu\to tt} \left[\frac{\ln^2(s_4/m^2)}{s_4}\right]_+ - 4C_F \ln\left(\frac{\mu_F^2}{m^2}\right) \left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ - \right. \\
& \left. - \zeta_2\, c_3^{uu\to tt} \left[\frac{1}{s_4}\right]_+ \right\}.
\end{aligned}
\tag{4.14}
$$

For the FCNC hadronic cross section $p(p_a) + \bar{p}(p_b) \to t(p_1) + t(p_2)$ we define $S = (p_a + p_b)^2$, $T = (p_a - p_1)^2$, and $U = (p_b - p_1)^2$, and note that $p_{u_a} = x_a p_a$, $p_{u_b} = x_b p_b$. The hadronic cross section is then given by

$$
\begin{aligned}
\sigma^{FCNC}_{p\bar{p}\to tt}(S) = {} & \int_{m^2-(S/2)\left(1+\sqrt{1-4m^2/S}\right)}^{m^2-(S/2)\left(1-\sqrt{1-4m^2/S}\right)} dT \int_{-S-T+2m^2}^{m^2+m^2S/(T-m^2)} dU \times \\
& \times \int_{(m^2-T)/(S+U-m^2)}^{1} dx_b \int_0^{x_b(S+U-m^2)+T-m^2} ds_4 \times
\end{aligned}
$$

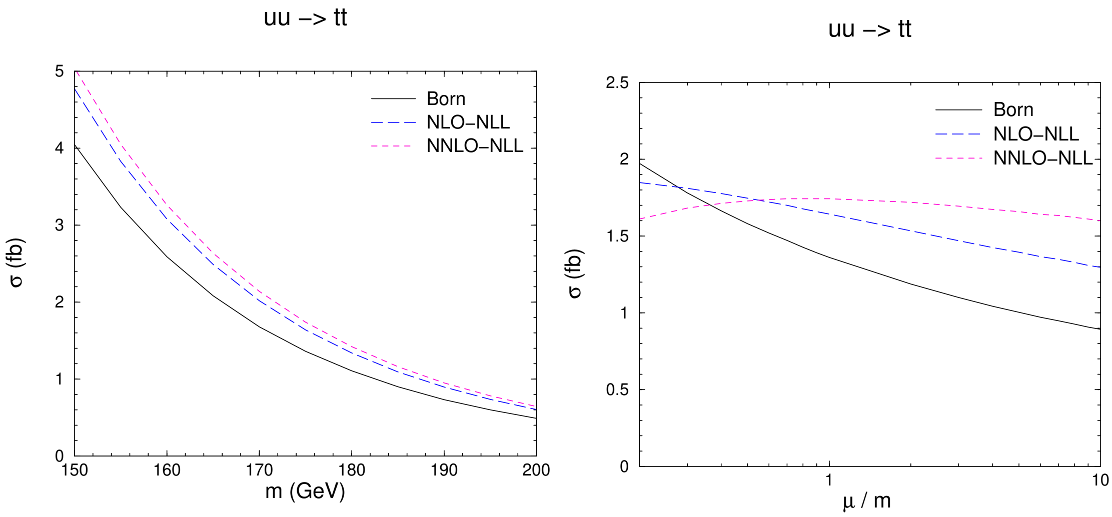

Figure 6: The Born, NLO-NLL, and NNLO-NLL cross sections for $uu \to tt$ in $p\bar{p}$ collisions with $\sqrt{S} = 1.96\,\text{TeV}$ and $\kappa_\gamma = \kappa_Z = 0.1$. Left: cross section versus top-quark mass at scale $\mu = m$; right: cross section versus the scale for $m = 175\,\text{GeV}$.

$$\times \frac{x_a x_b}{x_b S + T - m^2}\, \phi(x_a)\, \phi(x_b)\, \frac{d^2\hat{\sigma}_{uu\to tt}}{dt\, du} \tag{4.15}$$

with $x_a = \left[s_4 - x_b(U - m^2)\right] / (x_b S + T - m^2)$.

Our numerical results are presented in figure 6. The left frame presents the cross section for the process $uu \to tt$ versus top-quark mass at the Tevatron, with $\sqrt{S} = 1.96\,\text{TeV}$, at Born, NLO-NLL, and NNLO-NLL orders. One can see that the $\Delta\sigma_{\text{NLO-NLL}}$ and $\Delta\sigma_{\text{NNLO-NLL}}$ corrections are always positive for this case for $\mu = m$. The stabilisation of the NNLO-NLL cross section with respect to scale variation is even more pronounced for this process compared to the two previous cases as one can see in the right frame of figure 6. At $\mu = m = 175\,\text{GeV}$, we have for $\kappa_\gamma = \kappa_Z = 0.1$:

$$\sigma^{uu\to tt}_{\text{Born}} = 1.36\,\text{fb}\,, \qquad \Delta\sigma^{uu\to tt}_{\text{NLO-NLL}} = 0.28\,\text{fb}\,, \qquad \Delta\sigma^{uu\to tt}_{\text{NNLO-NLL}} = 0.10\,\text{fb}\,.$$

Taking into account the theoretical error due to the scale variation between $m/2$ and $2m$ one has

$$\sigma^{uu\to tt}_{\text{NNLO-NLL}} = 1.74^{+0.00}_{-0.02}\,\text{fb}\,.$$

We can investigate the magnitude of the cross section if one of the anomalous couplings is set to zero. If we use $\kappa_\gamma = 0.1$ and set $\kappa_Z = 0$ the NNLO-NLL cross section is 0.50 fb, while if we set $\kappa_\gamma = 0$ and use $\kappa_Z = 0.1$ the NNLO-NLL cross section is 0.38 fb.

We also note that we can have contributions from anomalous couplings involving the charm quark. The total cross section for the process $cc \to tt$ at the Tevatron, using $\kappa_{tc\gamma} = \kappa_{tcZ} = 0.1$, at $\mu = m = 175\,\text{GeV}$, is: $\sigma^{cc\to tt}_{\text{NNLO-NLL}} = 0.0044\,\text{fb}$. It is almost 3 orders of magnitude lower then the cross section for the $uu \to tt$ process since both c-quarks originate from the sea.

The total cross section at the Tevatron for the mixed process $uc \to tt$, using the value 0.1 for all anomalous couplings and $\mu = m = 175\,\text{GeV}$, is: $\sigma^{uc\to tt}_{\text{NNLO-NLL}} = 0.62\,\text{fb}$.

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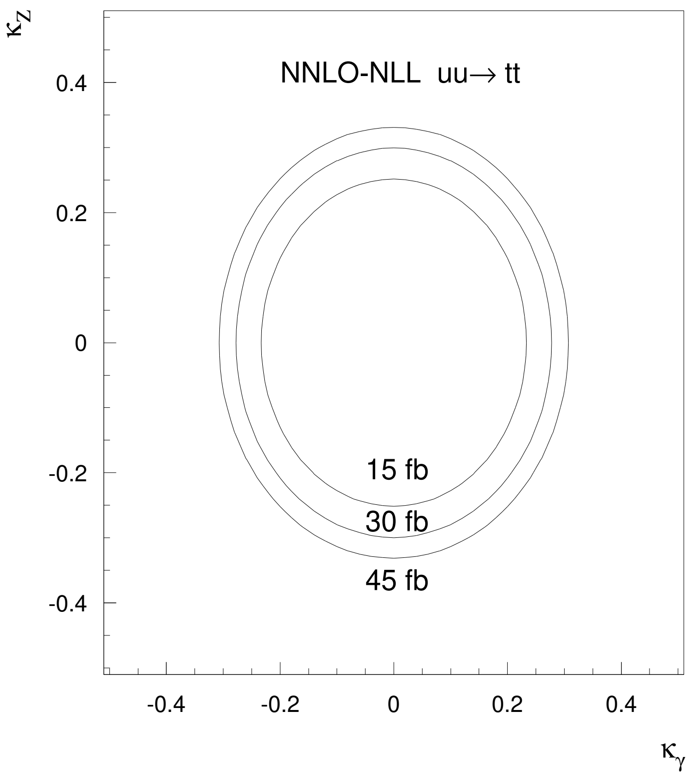

Figure 7: Tevatron reach in $\kappa_Z - \kappa_\gamma$ plane for the process $uu \to tt$ for cross section levels 15, 30, and 45 fb.

Also, we note that $\bar{t}\bar{t}$ FCNC production at the Tevatron has an equal cross section to tt production, so if it is included the total cross section is doubled.

Processes like same-sign top-quark production play a special role since tt final states give rise to same-sign leptons (SSL). We have estimated that potential $uu \to bbW^+W^+$ background can be safely neglected. Assuming $10\,\mathrm{fb}^{-1}$ integrated luminosity and 1% efficiency (including *b-tagging* and top-quark decay branchings) one should require 30 fb of signal in order to observe 3 signal events (to exclude signal at 95% CL according to Poisson statistics under the assumption that the background is negligible). We use this cross section as a rough idea on the order of the signal cross section to which Tevatron will be sensitive. Since the process $uu \to tt$ depends on *two* parameters, we present 15-30-45 fb contour levels of the $uu \to tt$ cross section in $\kappa_\gamma - \kappa_Z$ plane in figure 7. This figure, which gives a rough idea about the Tevatron reach, shows that the Tevatron sensitivity is just slightly better for the κ_γ coupling.

At this point, it is worth mentioning the present experimental limits on the κ_γ, κ_Z couplings. First, a limit on FCNC top-quark couplings has been established by the CDF collaboration in terms of the limit on the branching ratio of FCNC top-quark decay [21]: $\mathrm{Br}(t \to q\gamma) < 3.2\%$, $\mathrm{Br}(t \to qZ) < 33\%$ which corresponds to $\kappa_\gamma < 0.14$, $\kappa_Z < 0.69$ limit in the notation of eq. (1.1). A more stringent limit on $\kappa_\gamma < 0.10$ has been established by the L3 collaboration [22]. Finally, the best limits on κ_γ have been established by H1 ($\kappa_\gamma < 0.090$) [23, 24] and by ZEUS ($\kappa_\gamma < 0.058$) [25, 26, 27] collaborations. All the limits quoted above correspond to the notation of eq. (1.1). One can see that in light of the present experimental limits, the $uu \to tt$ process is unlikely to be observed at the Tevatron unless $uu \to tt$ production involves chromomagnetic flavor-changing current, which is beyond the scope of the present paper. In order to estimate Tevatron sensitivity to $p\bar{p} \to tZ$, $p\bar{p} \to t\gamma$ processes one should study in details all potential backgrounds which is, again, beyond the scope of our paper.

5. $eu \to et$

The last process we present is single-top production at HERA [25, 26, 27, 23, 24]. We have previously studied this process in ref. [12] where we calculated the NLO soft corrections with $tq\gamma$ couplings. Here we extend our previous calculation by also including tqZ couplings and NNLO-NLL soft corrections.

For the process $e(p_e) + u(p_u) \to e(p_f) + t(p_t)$, we define the kinematical invariants $s = (p_e + p_u)^2$, $t = (p_t - p_u)^2$, $u = (p_t - p_e)^2$, and $s_4 = s + t + u - m^2 - 2m_e^2$, with m the top-quark mass and m_e the electron mass. Note that near threshold $s_4 \to 0$. The respective tree-level Feynman diagrams are shown in figure 8.

The differential Born cross section is

$$\frac{d^2\hat{\sigma}_B^{eu\to et}}{dt\,du} = F_B^{eu\to et}\,\delta(s_4) \tag{5.1}$$

where

$$\begin{aligned} F^B_{eu\to et} = \frac{\pi\,\alpha^2}{m^2\,(s-m_e^2)^2}\Bigg[& \left\{-t\left[2m_e^4 + m^4 - 2s^2 + (2s+t)(2s-m^2-2m_e^2)\right] - 2m_e^2 m^4\right\} \times \\ & \times\left\{\frac{8\kappa_\gamma^2}{t^2} + \frac{4\kappa_\gamma\kappa_Z(1-4\sin^2\theta_W)}{\sin\theta_W\,\cos\theta_W\,t\,(t-m_Z^2)} + \right. \\ & \left. + \frac{\kappa_Z^2(1-4\sin^2\theta_W + 8\sin^4\theta_W)}{\sin^2\theta_W\,\cos^2\theta_W\,(t-m_Z^2)^2}\right\} \\ & - \frac{\kappa_Z^2 m_e^2(-2m^4 + tm^2 + t^2)}{\sin^2\theta_W\,\cos^2\theta_W\,(t-m_Z^2)^2}\Bigg], \end{aligned} \tag{5.2}$$

with θ_W the weak-mixing angle, $\kappa_\gamma \equiv \kappa_{tu\gamma}$, and $\kappa_Z \equiv \kappa_{tuZ}$.

The NLO-NLL corrections for $eu \to et$ are

$$\frac{d^2\hat{\sigma}^{(1)}_{eu\to et}}{dt\,du} = F_B^{eu\to et}\frac{\alpha_s(\mu_R^2)}{\pi}\left\{c_3^{eu\to et}\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ + c_2^{eu\to et}\left[\frac{1}{s_4}\right]_+ + c_1^{eu\to et}\delta(s_4)\right\}. \tag{5.3}$$

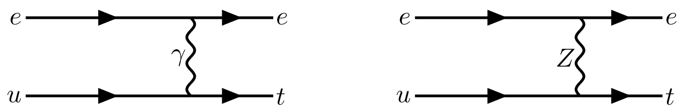

Figure 8: Tree-level Feynman diagrams for the process $eu \to et$.

Here $c_3^{eu\to et}=2C_F$,

$$\begin{aligned} c_2^{eu\to et} &= 2{\Gamma'}_S^{(1)} - C_F\left[1+2\ln\left(\frac{-u+m_e^2}{m^2}\right)+\ln\left(\frac{\mu_F^2}{s}\right)\right] \\ &\equiv T_2^{eu\to et} - C_F\ln\left(\frac{\mu_F^2}{m^2}\right), \end{aligned} \tag{5.4}$$

where ${\Gamma'}_S^{(1)}=C_F\ln[(m^2-t)/(\sqrt{s}m)]$ is the one-loop soft anomalous dimension and $T_2^{eu\to et}$ is the scale-independent part of $c_2^{eu\to et}$, and

$$c_1^{eu\to et} = \left[-\frac{3}{4}+\ln\left(\frac{-u+m_e^2}{m^2}\right)\right] C_F\ln\left(\frac{\mu_F^2}{m^2}\right) \tag{5.5}$$

is the scale-dependent part of the $\delta(s_4)$ terms.

The NNLO-NLL corrections for $eu\to et$ are

$$\begin{aligned} \frac{d^2\hat{\sigma}^{(2)}_{eu\to et}}{dt\,du} &= F_B^{eu\to et}\frac{\alpha_s^2(\mu_R^2)}{\pi^2}\times \\ &\times\left\{\frac{1}{2}\left(c_3^{eu\to et}\right)^2\left[\frac{\ln^3(s_4/m^2)}{s_4}\right]_+ + \right. \\ &+\left[\frac{3}{2}c_3^{eu\to et}\,c_2^{eu\to et}-\frac{\beta_0}{4}c_3^{eu\to et}\right]\left[\frac{\ln^2(s_4/m^2)}{s_4}\right]_+ + \\ &+\left[c_3^{eu\to et}\,c_1^{eu\to et}+C_F^2\ln^2\left(\frac{\mu_F^2}{m^2}\right)-2C_F T_2^{eu\to et}\ln\left(\frac{\mu_F^2}{m^2}\right)+\right. \\ &\left.+\frac{\beta_0}{4}c_3^{eu\to et}\ln\left(\frac{\mu_R^2}{m^2}\right)-\zeta_2\left(c_3^{eu\to et}\right)^2\right]\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ + \\ &+\left[-C_F\ln\left(\frac{\mu_F^2}{m^2}\right)c_1^{eu\to et}-\frac{\beta_0}{4}C_F\ln\left(\frac{\mu_F^2}{m^2}\right)\ln\left(\frac{\mu_R^2}{m^2}\right)+\right. \\ &\left.\left.+C_F\frac{\beta_0}{8}\ln^2\left(\frac{\mu_F^2}{m^2}\right)-\zeta_2\,c_2^{eu\to et}\,c_3^{eu\to et}+\zeta_3\left(c_3^{eu\to et}\right)^2\right]\left[\frac{1}{s_4}\right]_+\right\}. \end{aligned} \tag{5.6}$$

For the FCNC hadronic cross section $e(p_e)+p(p_p)\to e(p_f)+t(p_t)$ we define $S=(p_e+p_p)^2$, $T=(p_t-p_p)^2$, and $U=(p_t-p_e)^2$, and note that $p_u=xp_p$. The hadronic cross section is then given by

$$\begin{aligned} \sigma^{FCNC}_{ep\to et}(S) &= \int_{T_{\min}}^{T_{\max}} dT\int_{-S-T+m^2+2m_e^2}^{U_{\max}} dU\int_0^{S+T+U-m^2-2m_e^2} ds_4\,\times \\ &\times\frac{x}{S+T-m^2-m_e^2}\,\phi(x)\,\frac{d^2\hat{\sigma}_{eu\to et}}{dt\,du} \end{aligned} \tag{5.7}$$

with

$$T_{\substack{\max\\ \min}} = m^2-\frac{(S-m_e^2)}{2S}\left[S+m^2-m_e^2\mp\sqrt{(S-m^2-m_e^2)^2-4m^2m_e^2}\right], \tag{5.8}$$

$U_{\max}=2m^2+m_e^2-T-S(m^2-T)/(S-m_e^2)-(S-m_e^2)m^2/(m^2-T)$, and $x=(s_4+m_e^2-U)/(S+T-m^2-m_e^2)$.

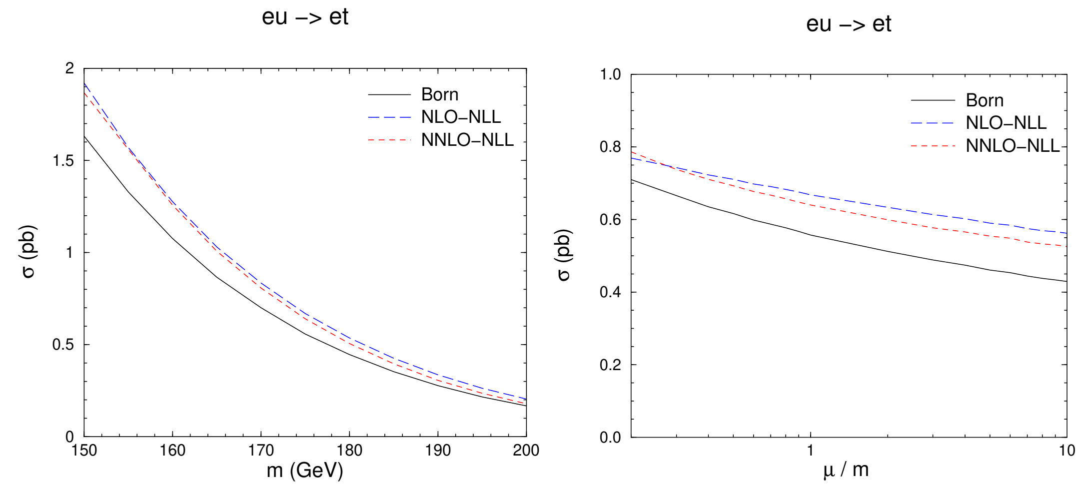

Figure 9: The Born, NLO-NLL, and NNLO-NLL cross sections for $eu \to et$ at HERA with $\sqrt{S} = 318\,\mathrm{GeV}$ and $\kappa_\gamma = \kappa_Z = 0.1$. Left: cross section versus top-quark mass at scale $\mu = m$; right: cross section versus the scale for $m = 175\,\mathrm{GeV}$.

Our numerical results are presented in figure 9. The left frame presents the cross section for the process $eu \to et$ versus top-quark mass at HERA, with $\sqrt{S} = 318\,\mathrm{GeV}$, at Born, NLO-NLL, and NNLO-NLL orders. One can see that the $\Delta\sigma_{\text{NLO-NLL}}$ corrections are positive while the $\Delta\sigma_{\text{NNLO-NLL}}$ corrections are negative for this case for $\mu = m$. The stabilisation of the NNLO-NLL cross section with respect to scale variation is shown in the right frame of figure 9. At $\mu = m = 175\,\mathrm{GeV}$, we have for $\kappa_\gamma = \kappa_Z = 0.1$:

$$\sigma^{eu\to et}_{\text{Born}} = 0.56\,\mathrm{pb}\,, \qquad \Delta\sigma^{eu\to et}_{\text{NLO-NLL}} = 0.11\,\mathrm{pb}\,, \qquad \Delta\sigma^{eu\to et}_{\text{NNLO-NLL}} = -0.03\,\mathrm{pb}\,.$$

Taking into account the theoretical error due to the scale variation between $m/2$ and $2m$ one has

$$\sigma^{eu\to et}_{\text{NNLO-NLL}} = 0.64^{+0.05}_{-0.04}\,\mathrm{pb}\,.$$

We note that almost all of the contribution comes from the κ_γ coupling. If we use $\kappa_\gamma = 0.1$ and set $\kappa_Z = 0$ the cross section remains practically unchanged (i.e. 0.64 pb at NNLO-NLL), while if we set $\kappa_\gamma = 0$ and use $\kappa_Z = 0.1$ the NNLO-NLL cross section is only 0.0018 pb.

In figure 10 we present contour levels for the $eu \to et$ process in $\kappa_\gamma - \kappa_Z$ plane. One of the contours is at 0.55 pb level which is the present HERA limit reported by H1 [23, 24] collaboration. The second contour is for 0.23 pb level which corresponds to the limit reported by ZEUS [27] collaboration. The third contour is for 0.1 pb level anticipating the improvement of the experimental limits for higher luminosities of the coming HERA II stage. Figure 10 clearly demonstrates that the HERA collider (in contrast to the $uu \to tt$ process at the Tevatron) is much more (about a factor of 40) sensitive to the κ_γ coupling than to the κ_Z one.

We also note that we can have contributions from anomalous couplings involving the charm quark. Using $\kappa_{tc\gamma} = \kappa_{tcZ} = 0.1$ the cross section at HERA for $ec \to et$, with $\mu = m = 175\,\mathrm{GeV}$, is $\sigma^{ec\to et}_{\text{NNLO-NLL}} = 0.0020\,\mathrm{pb}$, which is low due to the small sea charm-quark densities.

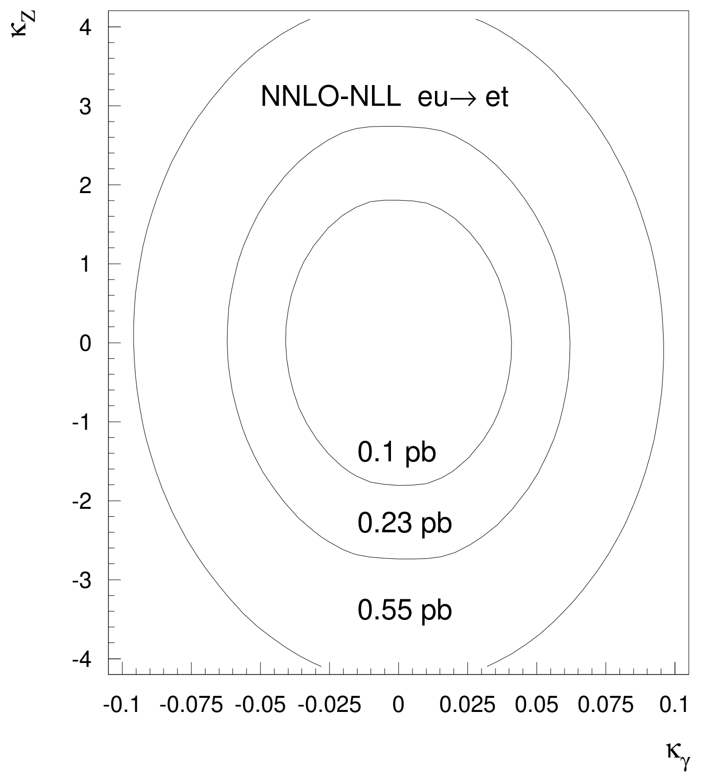

Figure 10: HERA reach in $\kappa_Z - \kappa_\gamma$ plane for the process $eu \to et$ for cross section levels 0.55, 0.23, and 0.1 pb.

In the case of $e\bar{t}$ production, involving the anti-top, the cross section is small, $\sigma^{e\bar{u}\to e\bar{t}}_{\text{NNLO-NLL}} = 0.0079\,\text{pb}$ at $\mu = m = 175\,\text{GeV}$, and thus asymmetrical to et production, in contrast to the other processes which we considered at the Tevatron. This is because at HERA the valence up-quark density can not contribute to $e\bar{t}$ production.

6. Conclusions

We have studied several FCNC processes at the Tevatron and HERA colliders involving top quark production via anomalous tqV couplings, with q an up or charm quark and V a photon or Z-boson. The partonic processes involved are $gu \to tZ$, $gu \to t\gamma$, $uu \to tt$, and $eu \to et$ as well as the corresponding ones with the up quark replaced by a charm quark. We have calculated NLO and NNLO soft-gluon corrections to these cross sections.

The Tevatron and HERA colliders provide a unique opportunity for precise study of FCNC couplings involving top-quark production. For some of these processes the uncertainty of the Born-level cross sections due to the choice of the QCD scale could be of the order of 100%. The search for FCNC single top-quark production at HERA is especially intriguing since a very recent experimental study of this process not only establishes a limit on the anomalous FCNC coupling but also presents an excess of events in the leptonic channel over the SM background (5 events observed while 1.31 ± 0.22 events are expected for SM background !) [24].

It is also worth mentioning that Tevatron and HERA are highly complementary in constraining FCNC top-quark couplings: HERA has the best potential in limiting the κ_γ coupling and is practically insensitive to κ_Z, while the Tevatron is sensitive to both κ_Z and κ_γ couplings. Therefore the combination of κ_γ limit from HERA and κ_Z limit from the Tevatron will be the most stringent limit in κ_γ - κ_Z plane.

The main outcome of this paper is that we have found that higher-order QCD corrections stabilize the factorization and renormalization scale dependence of the FCNC cross sections down to the level of about 10% or less, and provide modest to significant enhancements to the leading order result.

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